and J. C. Irvin were greatly appreciated. The writers are indebted to M. Humpel for helpful suggestions and the actual construction of the frequency multiplier.

Erling G. Wessel†
Norwegian Defence Research Establ.
Dept. for Telecommunication
Postboks 25
Kjeller, Norway
Robert J. Strain‡
Standard Telecommunication Labs.
Harlow, Essex, England

† Formerly with University of Colorado, Boulder.
‡ Formerly with University of Illinois, Urbana.

TM and TE Mode Surface Waves on Grounded, Anisotropic, Inhomogeneous, Lossless, Dielectric Slabs*

J. H. Richmond has given the WKB solutions for the field distribution of surface waves on inhomogeneous, isotropic, plane layers.[1] It is the purpose of this letter to extend his work to include a simple anistropy in the dielectric constant by considering a diagonalized relative permittivity tensor with components $\epsilon_x(x)$, $\epsilon_y(x)$, and $\epsilon_z(x)$. The geometry is the same as before[1] except that a perfectly conducting plane is now positioned at $x=0$. For easy reference we have used the same notation as Richmond, except where specified otherwise. Compactness in notation has been achieved by expressing the integrations from 0 to x and by considering the x variations outside the slab to be $\exp\{-\alpha(x-a)\}$.

The TM solutions for the x variations of the field components are given by

$$H_y = \begin{cases} \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{g(x)}{g(a)} \cdot \dfrac{\cos R(x)}{\cos R(a)}, & \text{in region II,} \end{cases} \tag{1}$$

$$E_z = \begin{cases} \dfrac{j\alpha}{\omega\epsilon_0} \cdot \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{j\alpha}{\omega\epsilon_0} \cdot \dfrac{g(a)}{g(x)} \cdot \dfrac{\sin R(x)}{\sin R(a)}, & \text{in region II,} \end{cases} \tag{2}$$

where

$$g(x) = \left[\frac{\epsilon_z(x)}{r(x)}\right]^{1/2}, \tag{3}$$

$$r(x) = \left[\frac{\epsilon_z(x)}{\epsilon_x(x)} \cdot (k^2\epsilon_x(x) - h^2)\right]^{1/2}, \tag{4}$$

and

$$R(x) = \int_0^x r(x)dx. \tag{5}$$

It is observed that H_y is normalized to unity at the air-slab interface. Also, we have considered the relative permeability of the slab to be unity.

The transcendental equation for the propagation constant h is given by

$$r(a) \tan R(a) = \alpha\epsilon_z(a). \tag{6}$$

Eq. (6), for a constant, scalar permittivity, reduces to (39) in a standard reference.[2]

The x variations for the TE modes are summarized as

$$E_y = \begin{cases} \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \left[\dfrac{q(a)}{q(x)}\right]^{1/2} \cdot \dfrac{\sin Q(x)}{\sin Q(a)}, & \text{in region II,} \end{cases} \tag{7}$$

$$H_z = \begin{cases} \dfrac{\alpha}{j\omega\mu_0} \exp\{-\alpha(x-a)\}, & \text{in region I,} \\ \dfrac{\alpha}{j\omega\mu_0} \cdot \left[\dfrac{q(x)}{q(a)}\right]^{1/2} \cdot \dfrac{\cos Q(x)}{\cos Q(a)}, & \text{in region II,} \end{cases} \tag{8}$$

where

$$q(x) = [k^2\epsilon_y(x) - h^2]^{1/2} \tag{9}$$

and

$$Q(x) = \int_0^x q(x)dx. \tag{10}$$

The determining equation for the propagation constant h is

$$q(a) \cot Q(a) = -\alpha, \tag{11}$$

which, for a constant, scalar permittivity, agrees with (46b) in a well-known text.[2]

It is to be noted that the solutions are valid only for slowly varying permittivities, or when

$$\left| \frac{r'(x)}{r^2(x)} - \frac{\epsilon_z'(x)}{r(x)\epsilon_z(x)} \right| \ll 2, \qquad \text{TM}, \tag{12}$$

$$\left| \frac{q'(x)}{q^2(x)} \right| \ll 2, \qquad \text{TE}, \tag{13}$$

where the prime denotes differentiation with respect to x. We find different restrictions on the TM and TE cases because the wave equations are different for the two cases, being given by

$$H_y'' - \frac{\epsilon_z'(x)}{\epsilon_z(x)} H_y' + \frac{\epsilon_z(x)}{\epsilon_x(x)} (k^2\epsilon_x(x) - h^2)H_y = 0, \qquad \text{TM}, \tag{14}$$

and

$$E_y'' + (k^2\epsilon_y(x) - h^2)E_y = 0, \qquad \text{TE}. \tag{15}$$

Expressions (12) and (13) can be verified by considering, in detail, standard WKB solutions of the Schroedinger equation.[3]

Conventionally,[1,4] we have neglected derivatives of $g(x)$ and $q(x)$ in finding E_z for the TM case and H_z for the TE case, respectively.

D. A. Holmes
Westinghouse Electric Corp.
Research and Development Center
Pittsburgh 35, Pa.

* Received August 26, 1963.

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[3] L. I. Schiff, "Quantum Mechanics," McGraw-Hill Book Company, Inc., New York, N. Y., pp. 184–193; 1955.

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Application of Corner Mirrors for Ultramicrowave Interferometers*

The Michelson type and Fabry-Perot type interferometers are often used in the field of ultramicrowaves; the latter replace conventional cavity resonators in the millimeter and submillimeter wave range.[1]

The main problem connected with these devices is the design of suitable mirrors with low loss and adequate reflectivity. So far, planar or spherical mirrors are used.[2] Adjustment of these mirrors is critical and tedious; hence, the use of optical collimation methods is recommended.[3] These difficulties could be remarkably reduced by application of metallic mirrors in the form of rectangular prism corners, *i.e.*, so-called corner mirrors (see Fig. 1).

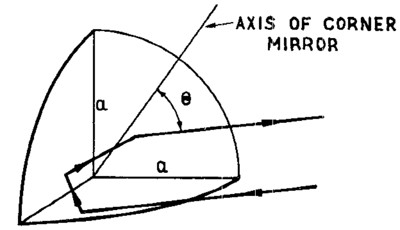

Fig. 1—Cubic corner mirror.

They take advantage of a very old principle of optics and have also been used sometimes in connection with radar techniques.[4] The adjustment of a corner mirror, which consists of three mutually perpendicular metallic plane mirrors, is in principle uncritical. For a correct design of a mirror, it is sufficient to secure the stable position of its peak. Then a beam of electromagnetic waves falling on a mirror at an angle of $\Theta < 35.26°$, after a triple reflection, will return antiparallelly to the falling beam. (For real mirrors having a finite ratio a/λ, a respectively smaller value of the angle Θ may be utilized.[1])

The interferometers utilizing corner mirrors could be designed in the form given in Figs. 2 and 3. The set in Fig. 2 differs from the conventional Michelson interferometer only by the type of the mirrors used, and, therefore, it does not require further discussion.

In the Fabry-Perot interferometer, one must secure suitable coupling with the cavity. For that purpose, one wall of the metallic corner should be half-transparent; a metallic perforated wall would be a good solution.[3] The Fabry-Perot interferometer with corner mirrors can be made in a number of variants (see Fig. 3). The set in Fig. 3(a) differs from that used in the past only by the type of mirrors used and additional

* Received August 16, 1963.

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